Rich Mixture Set, Process Preference, and Home Bias

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Abstract

Decision making often involves compounding of risks from different sources. Building on the Herstein-Milnor mixture set axiomatization of expected utility theory, we employ multiple mixture operators each modeling a source of risk to arrive at the definition of a rich mixture set, elements of which are rich lotteries. Our modeling framework enables a source-dependent weakening of the independence axiom as well as the reduction of compound lottery axiom. This yields a representation for preference over rich lotteries called source recursive expected utility (SREU). When there is consistent preference for the "same" lottery arising from different sources, SREU implies a preference for risk being resolved more decisively by the preferred source. We further show that an SREU investor always exhibits home bias when she consistently prefers risks arising from the domestic stock market over identically distributed risks from the foreign stock market.

Keywords: mixture sets, recursivity, rich lottery, source preference, reduction of compound lottery, process preference, home bias

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1 Introduction

The idea that equally distributed risks may be valued differently first appears in a question posed in Keynes (1921), *"If two probabilities are equal in degree, ought we, in choosing our course of action, to prefer that one which is based on a greater body of knowledge?"*[1] Such behavior, referred to as source preference, has been tested in a growing experimental literature since the early works of Heath and Tversky (1991) and Fox and Tversky (1995).[2] In this regard, Heath and Tversky further suggest that investors may sometimes be *"willing to forego the advantage of diversification and concentrate on a small number of companies with which they are presumably familiar"*.

In modeling a decision maker who is sensitive to sources of risk, it is necessary to consider the *process* of uncertainty resolution. When studying two-stage compound lotteries, Segal (1990) shows that if a model of decision making allows equally distributed risks arising from different stages to be valued differently, the reduction of compound lottery axiom (RCLA) fails to hold even under the assumptions of within-stage independence and recursive evaluation of compounding risks.[3] In other words, the decision maker may no longer be indifferent between two-stage lotteries and their actuarial reduction to simple lotteries.[4]

The setting of compound lotteries, defined by multiple stages of uncertainty resolution, including *subjective* compound lotteries residing on a product state space (such as those in Nau, 2006 and Ergin and Gul, 2009), is not sufficiently flexible for the analysis of source-sensitive decision making among lotteries with rich possibilities in the process of uncertainty resolution. For example, the same source may appear multiple times as uncertainty resolves in a lottery; two lotteries may differ by how different sources are ordered in their processes of uncertainty resolution.

In this paper, we offer a new domain for decision making to facilitate the modeling of choice behavior involving compounding of risks from distinct sources. Our

[1]This quote summarizes what Keynes wrote earlier in his treatise, "... in the first case we know that the urn contains black and white in equal proportions; in the second case the proportion of each colour is unknown, and each ball is as likely to be black as white. It is evident that in either case the probability of drawing a white ball is 1/2,..."; this unknown urn idea reappears in Ellsberg's (1961) two-urn thought experiment.

[2]These include Chew et al. (2008), Abdellaoui et al. (2011), Armantier and Treich (2016), and Chew et al. (2023), among others.

[3]Segal (1990) refers to these as mixture independence and compound independence axioms respectively.

[4]Following Halevy (2007), evidence of non-RCLA behavior has been reported in Abdellaoui et al. (2015), Chew et al. (2017), Dean and Ortoleva (2019), and Gillen et al. (2019), among others.

construction relies on Herstein and Milnor (1953) who offer an elegant axiomatization of expected utility theory based on a mixture set which is traceable to von Neumann and Morgenstern (1944). A *mixture set* $\mathcal{M}$ is endowed with a single mixture operation $\alpha a \oplus (1-\alpha)b$ which models a lottery delivering a with probability α and b with probability $1-\alpha$. To model source preference, we make use of a collection of mixture operators $\{\oplus_s\}_{s\in S}$ to define a *rich mixture set* with each operator modeling (and indexed by) a source of risk. Elements of a rich mixture set are referred to as *rich lotteries*. Abstracting away the state space, pure consequences, and the number of stages, rich mixture sets and rich lotteries provide a novel, parsimonious, and flexible structure to model the non-trivial *compounding of uncertainty* that arises from multiple sources of risk.

A key assumption on the standard mixture set which may be viewed as a precursor to RCLA, reflecting the irrelevance of the two-stage composition, is

$$\lambda[\mu a \oplus (1-\mu)b] \oplus (1-\lambda)b = \lambda\mu a \oplus (1-\lambda\mu)b$$

for each pair of a, b in $\mathcal{M}$ and each $\lambda, \mu \in [0,1]$. Rather than assuming overall RCLA regardless of different sources of risk, we require irrelevance of two-stage composition for risks arising only from the same source as part of the definition of source-specific mixture operators. Chew et al. (2023) provide support for our source-based decomposition of overall RCLA among their main experimental findings involving over 3000 subjects. Around two thirds of the subjects exhibit a joint non-neutral attitude towards different sources of risk as well as compounding of risks relative to the benchmark simple lottery.

In our axiomatization, we retain the ordering and continuity axioms of Herstein and Milnor (1953) and relax the independence axiom in rich mixture sets by requiring it to hold only for risks arising from the same source. In this way, we are led to an axiomatization of a *source recursive expected utility* (SREU) in our main Theorem. In general, our approach allows for a broad range of source-based process preference and offers a tool to characterize them. In particular, when there is a consistent preference for the "same" lottery being resolved by one source (the preferred source) rather than by another, SREU brings out a preference for risk being resolved more decisively by the preferred source, where decisiveness is in the sense of "more" uncertainty being resolved by a particular source (Proposition 1, Section 4). We offer an account of home bias in international finance (Feldstein and Horioka, 1980; French and Poterba, 1991) by drawing on a strict preference for the source of risk arising from the domestic stock market and and a perception of the internationally diversified portfolio in terms of a rich lottery (Proposition 2, Section 5).

The rest of the paper is organized as follows. The formal definition of a rich mixture set is provided in Section 2. Axioms and the representation theorem are provided in Section 3. Our applications of SREU to model process preference and to account for home bias are discussed respectively in Sections 4 and 5. Section 6 concludes.

2 Rich Mixture Set

The axioms in Herstein and Milnor's (1953) characterization of expected utility theory are based on a single mixture operation $\alpha a \oplus (1-\alpha)b$ defined on a mixture set $\mathcal{M}$ which formalizes the earlier construction in von Neumann and Morgenstern (1944). The preceding discussion concerning source preference in the introduction motivates us to make use of a collection of such mixture operators $\{\oplus_s\}_{s\in S}$ to define a rich mixture set formally below.

Definition 1. *Call $(\mathcal{M}, \{\oplus_s\}_{s\in S})$ a* ***rich mixture set*** *if for each $s \in S$, there is a mixture operator $\oplus_s : [0,1] \times \mathcal{M} \times \mathcal{M} \to \mathcal{M}$ with corresponding mixture operation $\mu a \oplus_s (1-\mu)b$ satisfying:*

1. $1a \oplus_s (1-1)b = a$,
2. $\mu a \oplus_s (1-\mu)b = (1-\mu)b \oplus_s \mu a$,
3. $\lambda[\mu a \oplus_s (1-\mu)b] \oplus_s (1-\lambda)b = \lambda\mu a \oplus_s (1-\lambda\mu)b$,

for any $a, b \in \mathcal{M}$ and any $\mu, \lambda \in [0,1]$.

Figure 1 below illustrates how an element $c = \alpha a \oplus_s (1-\alpha)b$ of the rich mixture set $\mathcal{M}$ can be interpreted as a lottery with α chance to receive a and $1-\alpha$ chance to receive b where the risk arises from source $s \in S$. As illustrated, a and b are themselves elements of $\mathcal{M}$ and therefore the mixture c may contain further information about how uncertainty arises within a and b, i.e., from sources u and v as illustrated in Figure 1. An element of $\mathcal{M}$ is referred to as a rich lottery given the richness of the information contained.

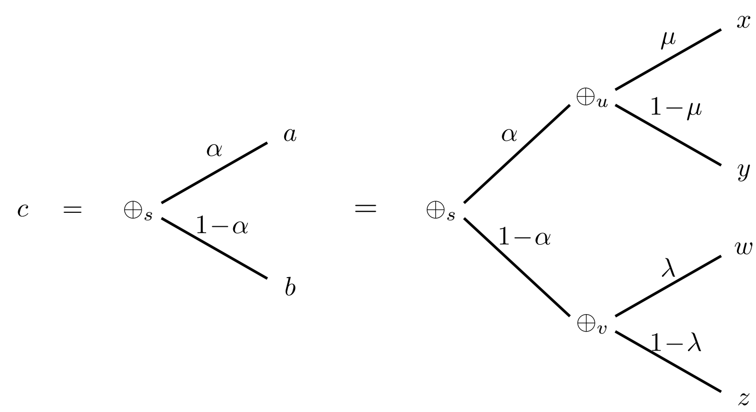

Figure 1: Example of Rich Lottery

In Figure 1, should s, u, and v represent the same source, RCLA would apply. This results in a one-stage lottery which delivers the outcomes x, y, w, and z with probabilities $\alpha\mu$, $\alpha(1-\mu)$, $(1-\alpha)\lambda$, and $(1-\alpha)(1-\lambda)$ respectively all arising from this source. In general, rich lotteries incorporate and bring out explicitly a new ingredient in decision making, i.e., the decision maker's *uncertainty perception*. That is to say, a rich lottery specifies how a decision maker perceives the process of uncertainty resolution from multiple sources of risk.

3 Source Recursive Expected Utility

We impose the following axioms on $\succsim$ starting with the standard axioms of ordering and continuity.

Axiom 1. *(Ordering) The preference ordering $\succsim$ is complete and transitive.*[5]

Axiom 2. *(Continuity) For any $a, b, c \in \mathcal{M}$ and for any $s \in S$, $\{\alpha : \alpha a \oplus_s (1-\alpha)b \succsim c\}$ and $\{\alpha : c \succsim \alpha a \oplus_s (1-\alpha)b\}$ are closed.*

The following axiom is a relaxation of Herstein and Milnor's (1953) independence axiom.

Axiom 3. *(Source Independence) For any $a, b, c \in \mathcal{M}$ and for any $s \in S$, $a \sim b$ implies $\frac{1}{2}a \oplus_s \frac{1}{2}c \sim \frac{1}{2}b \oplus_s \frac{1}{2}c$*

[5]We say $\succsim$ is complete if $a \succsim b$ or $b \succsim a$ for all $a, b \in \mathcal{M}$. We say $\succsim$ is transitive if $a \succsim b$ and $b \succsim c$ implies $a \succsim c$ for all $a, b, c \in \mathcal{M}$.

To facilitate comparison with an expected utility (EU) maximizer, observe that the standard independence axiom may be stated as follows: for any $a, b, c \in \mathcal{M}$ and for any $s, t \in S$, $a \sim b$ implies $\frac{1}{2}a \oplus_s \frac{1}{2}c \sim \frac{1}{2}b \oplus_t \frac{1}{2}c$. This is weakened by source independence which requires independence only for risks arising from the same source. Source independence enables our departure from RCLA while the standard independence subsumes RCLA.

The specification for SREU is formally stated below.

Definition 2. *We say the preference ordering $\succsim$ has a source recursive expected utility representation if there exists a utility function $U : \mathcal{M} \to \mathbb{R}$ representing $\succsim$, i.e., $U(a) \geq U(b)$ if and only if $a \succsim b$ for each $a, b \in \mathcal{M}$, with its range containing 0 and a strictly increasing continuous transformation function $T_s : \mathbb{R} \to \mathbb{R}$ with $T_s(0) = 0$ for each $s \in S$ such that*

$$U(\alpha a \oplus_s (1-\alpha)b) = T_s^{-1}(\alpha T_s \circ U(a) + (1-\alpha)T_s \circ U(b))$$

for all $a, b \in \mathcal{M}$ and all $0 \leq \alpha \leq 1$.

An SREU representation of $\succsim$ refers to a system of utility function and transformation functions $(U, \{T_s\}_{s \in S})$. To understand the recursive nature of SREU, consider $c = \alpha a \oplus_s (1-\alpha)b$ with $a = \mu x \oplus_u (1-\mu)y$ and $b = \lambda w \oplus_v (1-\lambda)z$ mentioned in Figure 1 earlier. Given utilities $U(x), U(y), U(w)$, and $U(z)$, the utility of c can be arrived at recursively by deriving $U(a)$ and $U(b)$ followed by $U(c)$:

$$\begin{aligned} U(a) &= T_u^{-1}(\mu T_u \circ U(x) + (1-\mu)T_u \circ U(y)), \\ U(b) &= T_v^{-1}(\lambda T_v \circ U(w) + (1-\lambda)T_v \circ U(z)), \\ U(c) &= T_s^{-1}(\alpha T_s \circ U(a) + (1-\alpha)T_s \circ U(b)). \end{aligned}$$

We now present our representation theorem together with its proof for expository fluency.

Theorem. *A preference ordering $\succsim$ on a rich mixture set $\mathcal{M}$ satisfies axioms 1-3 if and only if it has a SREU representation.*

Proof. For each $s \in S$, $(\mathcal{M}, \oplus_s)$ is a mixture set and the preference ordering $\succsim$ satisfies Herstein and Milnor's (1953) three axioms on $(\mathcal{M}, \oplus_s)$. This yields a von Neumann-

Morgenstern utility function $U_s : \mathcal{M} \to \mathbb{R}$ representing $\succsim$ such that

$$U_s(\alpha a \oplus_s (1-\alpha)b) = \alpha U_s(a) + (1-\alpha)U_s(b)$$

for all $a, b \in \mathcal{M}$ and all $0 \leq \alpha \leq 1$. Fix an arbitrary $s^* \in S$ and choose U_{s^*} such that there exists an element $a^* \in \mathcal{M}$ with $U_{s^*}(a^*) = 0$. Let $U = U_{s^*}$ be the utility function in the SREU representation with the transformation function for source s^* being the identity function $T_{s^*}(x) = x$. For each $s \in S$, choose U_s such that $U_s(a^*) = 0$. For each $s \in S$, we can define T_s to be a function such that $T_s \circ U(a) = U_s(a)$ for each $a \in \mathcal{M}$. T_s is uniquely defined on the image set of $U(\cdot)$, i.e., the interval $I = U(\mathcal{M})$ with $T_s(0) = 0$ and strictly increasing on this range since each U_s represents the same preference ordering as U. Furthermore, T_s is also continuous on I. To see why, consider $a, b, c, d \in \mathcal{M}$ such that $a \succsim b, c \succsim d$, $b \sim \alpha^* a \oplus_s (1-\alpha^*)d$, and $c \sim \alpha^* a \oplus_{s^*} (1-\alpha^*)d$ for the same $\alpha^* \in [0,1]$. Let $\{\alpha_n\}$ be any sequence of real numbers in $[0,1]$ converging to α^*. Under Axiom 2, $\{U_s(\alpha_n a \oplus_s (1-\alpha_n)d)\}$ converges to $U_s(b)$ and $\{U(\alpha_n a \oplus_{s^*} (1-\alpha_n)d)\}$ converges to $U(c)$. Since a, b, c, d are chosen arbitrarily, T_s is continuous on I.

The converse is straightforward. □

Observing that our SREU representation given by $(U, \{T_s\}_{s\in S})$ generalizes EU in accommodating the additional mixture operations, it is instructive to discuss its uniqueness property in relation to the uniqueness class of EU defined by positive affine transformations. Fixing the utility function U, it is clear that the transformation functions of an SREU representation are unique up to multiplication by a positive scalar on the effective domain of $U(\mathcal{M})$. Considering another utility function $\tilde{U} = T \circ U$ also representing $\succsim$ transformed by an arbitrary strictly increasing continuous function, it is clear that $(\tilde{U}, \{T_s \circ T^{-1}\}_{s\in S})$ is also an SREU representation of $\succsim$. Notice that $T_s \circ U$ captures the *cardinal* information of risk attitude for source s.

4 Process Preference

When lotteries arising from source s are consistently preferred to identically distributed risks from source t, e.g., s may be more familiar than t, we say that source s is preferred to source t. This partial ordering over sources of risk is formally stated below.

Definition 3. *For any two sources $s, t \in S$, we say source s is weakly preferred to* *source t if it is the case that*

$$\alpha a \oplus_s (1-\alpha) b \succsim \alpha a \oplus_t (1-\alpha) b$$

for any $a, b \in \mathcal{M}$ and any $\alpha \in [0,1]$.

We can also define a strict preference for s over t by requiring a strict preference $\alpha a \oplus_s (1-\alpha) b \succ \alpha a \oplus_t (1-\alpha) b$ in the range of $\alpha \in (0,1)$ for any two rich lotteries $a, b \in \mathcal{M}$ between which decision maker is not indifferent.

Maintaining the assumptions of our Theorem, we could restrict $\alpha = \frac{1}{2}$ in Definition 3 to provide an equivalent definition of preference over sources. In light of this definition, the standard independence axiom mentioned in Section 3 is a combination of our source independence axiom and indifference between any pair of sources and will return us to an expected utility representation as in Herstein and Milnor (1953).

The preference among sources of risk can be captured by the relative curvature of their transformation functions under SREU. This sets the stage for our derivation of its implications on the decision maker's attitude towards the process of uncertainty resolution when risks are compounded. Notably, SREU embodies a preference for decisiveness of the preferred source (PDPS) in Statement B of the proposition below.

Proposition 1. *Under SREU, for any two sources of risk $s, t \in S$, the following statements are equivalent:*

A. Source s is weakly preferred to source t.

B. It is the case that

$$p_a a \oplus_s (1-p_a)\left[\frac{p_b}{1-p_a} b \oplus_t \frac{p_c}{1-p_a} c\right] \succsim (1-p_c)\left[\frac{p_a}{1-p_c} a \oplus_s \frac{p_b}{1-p_c} b\right] \oplus_t p_c c$$

for any $a, b, c \in \mathcal{M}$ such that $a \succsim b \succsim c$ or $c \succsim b \succsim a$, and for any $p_a, p_b, p_c \in (0,1)$ with $p_a + p_b + p_c = 1$.

C. The composite function $T_t \circ T_s^{-1}$ is weakly concave.

In the proof of Proposition 1, the equivalence between statements A and C follows from observing that source s is preferred to source t if and only if

$$T_t \circ T_s^{-1}(\alpha T_s \circ U(a) + (1-\alpha) T_s \circ U(b)) \geq \alpha T_t \circ U(a) + (1-\alpha) T_t \circ U(b)$$

for any $a, b \in \mathcal{M}$ and $\alpha \in [0,1]$. That Statement B implies Statement A follows directly from observing that the former reduces to the latter in the limiting case of $p_b = 0$. To complete the proof, we demonstrate, in the appendix, that Statement C implies Statement B.

The choice behavior in Statement B of Proposition 1, exemplifying a kind of PDPS, is illustrated using the figure below where $p_{\neg i} = 1 - p_i$ and $p_{i|\neg j} = p_i/(1-p_j)$ for $i, j \in \{a, b, c\}$. When b is intermediate in preference between a and c, or equivalently the utility distance between a and c is larger, uncertainty being resolved between a and c serves a more "decisive" role. The decision maker prefers to resolve this uncertainty in the first stage by the preferred source with the rich lottery on the left, rather than the rich lottery to the right associated with resolving more decisively with the less preferred source. Observe that a comparison between these two rich lotteries, differing only in the ordering of the two sources, cannot be modeled in the classical setting of two-stage compound lotteries.

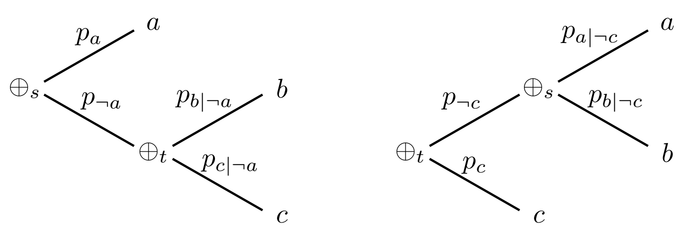

Figure 2: Preference for decisiveness of the preferred source

Statement B is also suggestive of the following assumption of (across-source) associativity: for any $s, t \in S$, any $a, b, c \in \mathcal{M}$, and for any $p_a, p_b, p_c \in (0,1)$ with $p_a + p_b + p_c = 1$, it is the case that

$$p_a a \oplus_s (1-p_a)\left[\frac{p_b}{1-p_a} b \oplus_t \frac{p_c}{1-p_a} c\right] \sim (1-p_c)\left[\frac{p_a}{1-p_c} a \oplus_s \frac{p_b}{1-p_c} b\right] \oplus_t p_c c \qquad (1)$$

As with the independence assumption or the assumption of indifference among all sources, imposing the above assumption on SREU also returns us to an EU representation. This observation pinpoints how we have relaxed RCLA in light of Rommeswinkel (2020) who, in a standard mixture set where there is only one source of risk, decomposes it into *reducibility*, i.e, $\alpha a \oplus (1-\alpha) a = a$ for any $\alpha \in [0,1]$ and $a \in \mathcal{M}$, and *associativity*, which is in the form of Expression (1) with a single mixture operation.

5 Home Bias

An investor faces a portfolio choice problem in allocating her total assets between the domestic stock market (D) and the foreign stock market (F). These are two sources of risk. A portfolio consisting of θ proportion of domestic stock is denoted by P_θ. For simplicity, suppose the returns of the two markets are independent and they yield the same return of \$1 or else \$0 with probability $\frac{1}{2}$.

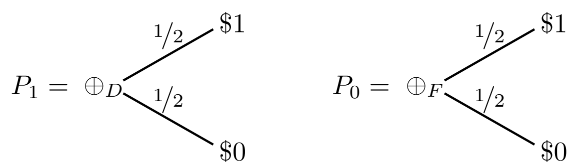

Figure 3: Domestic and foreign investments

If an SREU investor is indifferent between the two sources of risk, RCLA applies to the compounding of risks when diversifying between domestic and foreign stocks. Consequently, the utility of holding θ proportion of domestic stock is given by

$$U(P_\theta) = 0.25u(1) + 0.25u(\theta) + 0.25u(1-\theta) + 0.25u(0)$$

in which $u : [0,1] \to \mathbb{R}$ refers to the utility from investment returns. In this case, the transformation functions for both sources D and F can be fixed as the identity function $T(x) = x$ and omitted from the expression. Suppose u is strictly increasing and strictly concave, capturing investor's monotonicity towards investment returns and risk aversion, then her utility is maximized when she perfectly diversifies her portfolio, i.e., $\theta^* = \frac{1}{2}$. In other words, the investor does not exhibit home bias when she is indifferent between sources D and F.

Suppose otherwise that the SREU investor strictly prefers the domestic source of risk to the foreign source of risk. To proceed, let us further assume that the investor entertains the perception of the order of uncertainty resolution in the portfolio of holding θ proportion of domestic stock in terms of the following rich lottery.[6]

The utility $U(P_\theta)$ of the diversified portfolio for the SREU investor, as illustrated in Figure 4, can be derived as if the investor evaluates the perceived rich lottery recursively. The investor will first arrive at the certainty equivalents for the two 50-50

[6] The investor could also perceive the source of risk in the foreign stock market to be at the first stage. Under this alternative perception, we can derive the same Proposition 2.

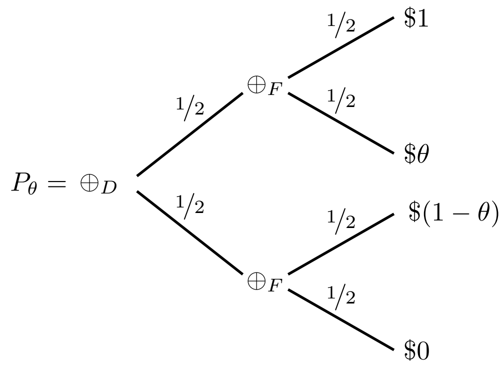

Figure 4: Perceived diversified portfolio

lotteries at the second stage, both of which arise from source F, with one delivering either \$1 or \$$\theta$ and the other delivering either \$$(1-\theta)$ or \$0. Their certainty equivalents c_θ and d_θ are given by $u(c_\theta) = T^{-1}(0.5T \circ u(1) + 0.5T \circ u(\theta))$ and $u(d_\theta) = T^{-1}(0.5T \circ u(1-\theta) + 0.5T \circ u(0))$ respectively, where T is the transformation function for source F. The investor then evaluates the diversified portfolio as a 50-50 lottery based on source D delivering c_θ and d_θ as outcomes:

$$U(P_\theta) = 0.5u(c_\theta) + 0.5u(d_\theta).$$

Under the assumption of a strict preference for domestic stock market, we have a strictly concave T. To arrive at the following proposition showing that such an investor always exhibits home bias, we further assume that u and T are differentiable.

Proposition 2. *Suppose the risk averse SREU investor facing the portfolio choice problem strictly prefers the domestic source of risk over the foreign source of risk. Then she exhibits home bias, i.e., only portfolio allocations such that $\theta^* \in (0.5, 1)$ can maximize $U(P_\theta)$.*

The proof of Proposition 2 is provided in the appendix.

6 Conclusion

In our approach to modeling choice behavior allowing for source preference under compounding of uncertainty, we make use of Herstein and Milnor's (1953) mixture set axiomatization to characterize SREU in which RCLA and independence applies within a single source of risk. Our approach allows for non-RCLA and non-independence behavior across sources due to the failure of associativity or non-indifference over sources when rich lotteries involve two or more sources. The setting of a rich mixture set provides a parsimonious and flexible analytical tool to model a broad range of choice phenomena involving multiple sources of risks.

Most theoretical treatments of source preference do not consider source-based process preference as is done in this paper. These include Chew and Sagi (2008), Gul and Pesendorfer (2015), and Cappelli et al. (2020). Both Chew and Sagi (2008) and Gul and Pesendorfer (2015) take, as primitive, a preference over Savagian acts and identify sources of risk in terms of collections of events. Cappelli et al. (2020) share with us the assumption of an exogenous set of sources. Unlike our approach, Cappelli et al. (2020) admit the possibility that the same sure outcome arising from different sources can be valued differently to capture "subjective prices".

A number of papers investigate the decision maker's preference over compound lotteries or other related domains without explicitly interpreting stages as different sources of uncertainty. Notably, Dillenberger (2010) captures a preference for immediate resolution of uncertainty based on a stage-invariant risk preference confining to negative certainty independence.[7] In the setting of temporal decision making, Kreps and Porteus (1978) captures a preference for early or late resolution of uncertainty with expected utilities potentially sensitive to different stages.[8] In this aspect, our model enables a characterization of preference over sources (in Definition 3) and the implied source-based process preference—a preference for decisiveness of preferred source (PDPS in Proposition 1).

In contrast with our approach, Rommeswinkel (2020) weakens RCLA by dropping reducibility while retaining associativity in order to characterize an entropy-based representation of procedural value. These developments set the stage for further theoretical development of process preference and further investigate the idea of recreational utility including a utility for gambling *per se* (von Neumann and Morgenstern, 1944).[9]

In the area of international finance, a satisfactory account of home bias has remained elusive.[10] Following Coval and Moskowitz (1999) and Huberman (2001), one

[7]Stage-invariance of risk preference is referred to as time neutrality in Dillenberger's paper.

[8]Moreover, many papers in this category attempt to explain ambiguity aversion by second-order belief, i.e., a belief over prior beliefs, and especially by decision maker's greater aversion to uncertainty arising from second-order belief (Segal, 1987; Nau, 2006; Klibanoff et al., 2005; Seo, 2009; Ergin and Gul, 2009; Evren, 2019)

[9]In their algebra of combining, von Neumann and Morgenstern highlight the role of the reduction property as in the opening paragraph of the Introduction in excluding a "utility of gambling".

[10]In his extensive comments as a discussant on Obstfeld and Rogoff (2001), Charles Engels views

direction involves seeking a preference-based explanation involving familiarity (see, e.g., Boyle et al., 2012; Solnik and Zuo, 2012; Asano and Osaki, 2020). Using SREU, we offer such an explanation for home bias in Proposition 2 of Section 5. By specifying the investor's uncertainty perception of the compounding risk in a diversified portfolio in terms of a rich lottery, our model enables the use of distinct utility functions capturing different risk attitudes towards assets across financial markets. In future research, an SREU-based approach may be added to the toolkit in addressing additional puzzles in financial markets that involve multiple sources of risk, e.g., non-participation, under-diversification, and excessive equity premium.

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Appendix

The proof of Proposition 1 makes use of the following lemma which is stated without proof.

Lemma 1. *Let $g : \mathbb{R} \to \mathbb{R}$ denote a strictly increasing, weakly concave function with $g(0) = 0$. If $x_1 x_2 \leq 0$, then $g(x_1 + x_2) \geq g(x_1) + g(x_2)$.*

Proof of Proposition 1: (C $\implies$ B) To simplify exposition, we only prove that the preference ordering holds when $U(b) = 0$. Apply the main Theorem to Statement B, yielding the following inequality to be shown based on Statement C.

$$T_s^{-1}\left(p_a T_s \circ U(a) + (1-p_a) T_s \circ T_t^{-1}\left(\frac{p_c}{1-p_a} T_t \circ U(c)\right)\right) \geq$$
$$T_t^{-1}\left(p_c T_t \circ U(c) + (1-p_c) T_t \circ T_s^{-1}\left(\frac{p_a}{1-p_c} T_s \circ U(a)\right)\right)$$

Letting x, and y, and $f(\cdot)$ denote $T_s \circ U(a)$, $T_t \circ U(c)$, and $T_t \circ T_s^{-1}(\cdot)$ respectively, the above inequality simplifies to:

$$f\Bigg(\underbrace{(1-p_a)f^{-1}\left(\frac{p_c}{1-p_a}y\right)}_{\text{Y}} + \underbrace{p_a x}_{\text{X}}\Bigg) \geq \underbrace{p_c y}_{\text{Y'}} + \underbrace{(1-p_c)f\left(\frac{p_a}{1-p_c}x\right)}_{\text{X'}} \tag{2}$$

By concavity of f, we have $f(\text{Y}) \geq (1-p_a) f \circ f^{-1}(\frac{p_c}{1-p_a}y) = \text{Y'}$ and $f(\text{X}) \geq \text{X'}$. Adding the two inequalities yields $f(\text{Y}) + f(\text{X}) \geq \text{Y'} + \text{X'}$, so that $f(\text{Y} + \text{X}) \geq \text{Y'} + \text{X'}$ by Lemma 1, as required to complete the proof. □

The lemma below, also stated without proof, is useful in the proof of Proposition 2.

Lemma 2. *Let $g : \mathbb{R} \to \mathbb{R}$ be strictly increasing and strictly convex. If $x_1 + y_1 = x_2 + y_2$ and $x_1 - y_1 > x_2 - y_2 \geq 0$, then $g(x_1) + g(y_1) > g(x_2) + g(y_2)$.*

Proof of Proposition 2: Observe that c_θ and d_θ, defined in Section 5, satisfy $\theta < c_\theta < 1$ and $0 < d_\theta < 1-\theta$ for each θ. Notice that $T \circ u(c_\theta) + T \circ u(d_\theta) = T \circ u(c_{1-\theta}) + T \circ u(d_{1-\theta})$ for each $\theta \in [0,1]$ and that the function $T \circ u(c_\theta) - T \circ u(d_\theta)$ of θ is strictly increasing with $T \circ u(c_0) - T \circ u(d_0) = 0$. Applying Lemma 2 for the strictly convex inverse transformation function T^{-1}, we have $U(P_\theta) > U(P_{1-\theta})$ for each $0.5 < \theta \leq 1$. Thus, any utility maximizing θ^* satisfies $0.5 \leq \theta^* \leq 1$.

The derivative for U_θ is given by

$$U'(\theta) = \frac{T' \circ u(\theta) u'(\theta)}{4T' \circ u(c_\theta)} - \frac{T' \circ u(1-\theta) u'(1-\theta)}{4T' \circ u(d_\theta)}.$$

Due to the strict concavity of T and u, the following inequalities hold:

$$U'(0.5) = \frac{T' \circ u(0.5) u'(0.5)}{4}\left(\frac{1}{T' \circ u(c_{0.5})} - \frac{1}{T' \circ u(d_{0.5})}\right) > 0;$$
$$U'(1) = 0.25(u'(1) - u'(0)) < 0.$$

Thus, θ^* cannot be 0.5 nor 1. This completes the proof. □